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HEAT AND MASS TRANSFER IN THE CASE OF SUBLIMATION IN A GAP BETWEEN ROTATING DISKS

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A solution is obtained of the problem of the temperature distribution in a sublimation flow in a slot.

The effect of sublimation on heat transfer in flat channels is considered in [1]. In what follows the problem of [1] is extended to the case of a sublimation flow in a gap between two rotating disks.

1. Consider a steady-state laminar flow of subliming vapor in a narrow slot between circular horizontal porous disks rotating with angular velocities ω_1 and ω_2. Let the lower disk be subjected to a constant, uniformly distributed heat flux with intensity q. The substance is sublimed from the upper disk at a constant rate w_s. The problem is solved in a cylindrical coordinate system with the axis z directed along the axis of the disks and the axis r along the radius of the slot. The coordinate origin is located on the axis of symmetry of the disks at an equal distance between them (Fig. 1). Assuming that the problem is one of rotational symmetry, we write the mass transfer and continuity equations for steady flow in the form

$$u\frac{du}{dr}+w\frac{du}{dr}-\frac{v^2}{r}=-\frac{dp}{dr}+\frac{1}{\mathrm{Re}}\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}+\frac{d^2u}{dz^2}-\frac{v}{r^2}\right),$$

$$u\frac{dv}{dr}+w\frac{dv}{dz}+\frac{uv}{r}=\frac{1}{\mathrm{Re}}\left(\frac{d^2v}{dr^2}+\frac{1}{r}\frac{dv}{dr}+\frac{d^2v}{dz^2}-\frac{v}{r^2}\right), \qquad (1)$$

$$u\frac{dw}{dr}+w\frac{dw}{dz}=-\frac{dp}{dz}+\frac{1}{\mathrm{Re}}\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}+\frac{d^2w}{dz^2}\right),$$

$$\frac{du}{dr}+\frac{u}{r}+\frac{dw}{dz}=0.$$

In reducing system (1) to dimensionless form, the components of the velocity vector u, v, w were referred to w_s, the pressure to ρw_s^2, and the coordinates to the characteristic geometrical parameter h. The boundary conditions are

$$u=0,\ \ v=\omega_1 r,\ \ w=w_1 \ \ \text{at} \ \ z=-1;$$

$$u=0,\ \ v=\omega_2 r,\ \ w=w_s \ \ \text{at} \ \ z=1.$$

A solution of this system will be sought in the form

$$u=-\frac{r}{2}f'(z),\ \ v=r\varphi(z),\ \ w=f(z), \qquad (2)$$

where f, f, φ are dimensionless functions.

Substitution of Eq. (2) into Eq. (1) gives the system of ordinary differential equations

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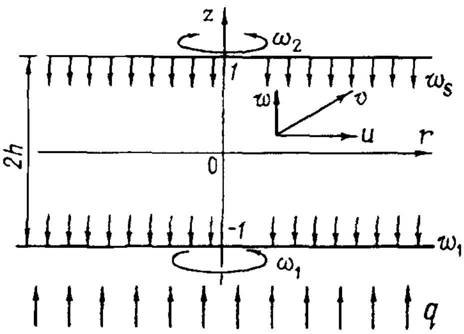

Fig. 1. Schematic diagram of flow in slot.

$$f''' + \mathrm{Re}\left(\frac{1}{2}f'^2 - ff'' - 2\varphi^2\right) = -\frac{2}{r}\mathrm{Re}\,\frac{dp}{dr},$$
$$\varphi'' - \mathrm{Re}\,(f\varphi' - \varphi f') = 0\,,\quad f'' - ff'\,\mathrm{Re} = \mathrm{Re}\,\frac{dp}{dz}. \tag{3}$$

In this case the continuity equation is satisfied identically.

Using cross-differentiation with respect to z and r to exclude the pressure p from the 1st and 3d equation in (3), we arrive at the system of two ordinary differential equations in the unknown functions f and u

$$f^{\mathrm{IV}} + \mathrm{Re}\,(-ff''' - 4\varphi) = 0\,,\quad \varphi'' - \mathrm{Re}\,(f\varphi' - \varphi f') = 0\,. \tag{4}$$

In dimensionless form the boundary conditions are

$$f'(-1) = 0\,,\quad \varphi(-1) = \alpha_1\,,\quad f(-1) = \beta \quad \text{at} \quad z = -1\,;$$
$$f'(1) = 0\,,\quad \varphi(1) = \alpha_2\,,\quad f(1) = 1 \quad \text{at} \quad z = 1\,. \tag{5}$$

A solution of Eq. (4) is sought in the form of the series (Re << 1)

$$f = \sum_{n=0}^{\infty} f_n\,\mathrm{Re}\,,\quad f' = \sum_{n=0}^{\infty} f_n'\,\mathrm{Re}\,,\quad \varphi = \sum_{n=0}^{\infty} \varphi_n\,\mathrm{Re}\,.$$

Using the method of successive approximations and taking only the zeroth and first approximations, instead of (4) we write

$$f_0^{\mathrm{IV}} + \mathrm{Re}\,f_1^{\mathrm{IV}} + \mathrm{Re}\,(-f_0 f_0''' - 4\varphi_0) = 0\,,\quad \varphi_0'' + \mathrm{Re}\,\varphi_1'' - \mathrm{Re}\,(f_0\varphi_0' - \varphi_0 f_0') = 0\,. \tag{6}$$

The zeroth approximation allows us to obtain

$$f_0^{\mathrm{IV}} = 0\,,\quad f_0''' = C_1\,,\quad f_0'' = C_1 z + C_2\,,\quad f_0' = C_1\frac{z^2}{2} + C_2 z + C_3\,,$$
$$f_0 = C_1\frac{z^3}{6} + C_2\frac{z^2}{2} + C_3 z + C_4\,;\quad \varphi_0'' = 0\,,\quad \varphi_0' = k_1\,,\quad \varphi_0 = k_1 z + k_2\,, \tag{7}$$

where $C_1 - C_4$, k_1, k_2 are integration constants.

In the first approximation system (6) gives the quadratures of the functions f_1 and φ_1:

$$f_1^{\mathrm{IV}} = f_0 f_0''' + 4\varphi_0\,,\quad \varphi_1^{\mathrm{IV}} = f_0\varphi_0' - \varphi_0 f_0'\,. \tag{8}$$

Substitution of expressions (7) into (8) gives

$$f_1^{IV} = \left(C_1 \frac{z^3}{6} + C_2 \frac{z^2}{2} + C_3 z + C_4\right) C_1 + (k_1 z + k_2)\,,$$

$$\varphi_1'' = \left(C_1 \frac{z^3}{6} + C_2 \frac{z^2}{2} + C_3 z + C_4\right) k_1 - (k_1 z + k_2) \left(C_1 \frac{z^2}{2} + C_2 z + C_3\right). \tag{9}$$

Integration of (9) leads to the relations

$$f_1''' = C_1^2 \frac{z^4}{24} + C_1 C_2 \frac{z^3}{6} + C_1 C_3 \frac{z^2}{2} + C_1 C_4 z + 2k_1 \frac{z^2}{1} + 4k_2 - z + n_1\,,$$

$$f_1'' = C_1^2 \frac{z^5}{120} + C_1 C_2 \frac{z^4}{24} + C_1 C_3 \frac{z^3}{6} + C_1 C_4 \frac{z^2}{2} + 2k_1 \frac{z^2}{3} + 2k_2 \frac{z^2}{1} + n_1 z + n_3\,,$$

$$f_1' = C_1^2 \frac{z^6}{720} + C_1 C_2 \frac{z^5}{120} + C_1 C_3 \frac{z^4}{24} + C_1 C_4 \frac{z^3}{6} + k_1 \frac{z^2}{6} + 2k_2 \frac{z^3}{3} + n_1 \frac{z^2}{2} + n_2 z + n_3\,, \tag{10}$$

$$f_1 = C_1^2 \frac{z^7}{5040} + C_1 C_2 \frac{z^6}{720} + C_1 C_3 \frac{z^5}{120} + C_1 C_4 \frac{z^4}{24} + k_1 \frac{z^5}{30} + k_2 \frac{z^4}{6} + n_1 \frac{z^3}{6} + n_2 \frac{z^2}{2} + n_3 z + n_4\,.$$

In a similar way, for the function φ we have

$$\varphi_1' = C_1 k_1 \frac{z^4}{24} + C_2 k_1 \frac{z^3}{6} + C_3 k_1 \frac{z^2}{2} + C_4 k_1 z - k_1 C_1 \frac{z^4}{8} - k_1 C_2 \frac{z^3}{3} -$$

$$- k_1 C_3 \frac{z^2}{2} - C_1 k_2 \frac{z^3}{6} - C_2 k_2 \frac{z^2}{2} - C_3 k_2 z + p_1\,, \tag{11}$$

$$\varphi_1 = C_1 k_1 \frac{z^5}{120} + C_2 k_1 \frac{z^4}{24} + C_3 k_1 \frac{z^3}{6} + C_4 k_1 \frac{z^2}{2} - k_1 C_1 \frac{z^5}{40} - k_1 C_2 \frac{z^4}{12} -$$

$$- k_1 C_3 \frac{z^3}{6} - C_1 k_2 \frac{z^4}{24} - C_2 k_2 \frac{z^3}{6} - C_3 k_2 \frac{z^2}{2} + p_1 z + p_2\,.$$

In systems (10) and (11) the integration constants $n_1 - n_4$, p_1, p_2 are determined under boundary conditions (5). In this case, use is made of the equations $f_1' = f_1'(z)$ and $f_1 = f_1(z)$ in system (10) and $\varphi_1 = \varphi_1(z)$ in (11).

Thus, the solution of problem (1) and (2) is expressed in the form

$$u = -\left(C_1 \frac{z^3}{2} + C_2 z + C_3\right) - \mathrm{Re}\left(C_1^2 \frac{z^6}{720} + C_1 C_2 \frac{z^5}{120} + C_1 C_3 \frac{z^4}{24} + \right.$$

$$\left. + C_1 C_4 \frac{z^3}{6} + k_1 \frac{z^4}{6} + 2k_2 \frac{z^3}{3} + n_1 \frac{z^2}{2} + n_2 z + n_3\right),$$

$$v = k_1 z + k_2 + \mathrm{Re}\left(C_1 k_1 \frac{z^5}{120} + C_2 k_1 \frac{z^4}{24} + C_3 k_1 \frac{z^3}{6} + C_4 C_1 \frac{z^2}{2} - \right.$$

$$\left. - k_1 C_1 \frac{z^5}{40} - k_1 C_2 \frac{z^4}{12} - k_1 C_3 \frac{z^3}{6} - C_1 k_2 \frac{z^4}{24} - C_2 k_2 \frac{z^3}{6} - C_3 k_2 \frac{z^2}{2} + p_1 z + p_2\right), \tag{12}$$

$$w = C_1 \frac{z^3}{6} + C_2 \frac{z^2}{2} + C_3 z + C_4 + \mathrm{Re} \left(C_1^2 \frac{z^7}{5040} + C_1 C_2 \frac{z^6}{720} + C_1 C_3 \frac{z^5}{120} + \right.$$

$$\left. + C_1 C_3 \frac{z^5}{120} + C_1 C_4 \frac{z^4}{24} + k_1 \frac{z^5}{30} + k_2 \frac{z^4}{6} + n_1 \frac{z^3}{6} + n_2 \frac{z^2}{2} + n_3 z + n_4 \right) .$$

For determination of the integration constants $C_1 - C_4$, k_1, k_2, $n_1 - n_4$, p_1, and p_2, boundary conditions (5) were expanded into power series. In the zeroth approximation we have

$$f_0'(-1) = 0 , \ \varphi_0(-1) = \alpha_1 , \ f_0(-1) = \beta ;$$
$$f_0'(1) = 0 , \ \varphi_0(1) = \alpha_2 , \ f_0(1) = 1 . \tag{13}$$

The first approximation gives

$$f_1'(-1) = \varphi_1(-1) = f_1(-1) = 0 ; \ f_1'(1) = \varphi_1(1) = f_1(1) = 0 . \tag{14}$$

As a result, using (13), we obtain from (7)

$$C_1 = \frac{3}{2}(\beta - 1) , \ C_2 = 0 , \ C_3 = \frac{3}{4}(1 - \beta) , \ C_4 = \frac{1}{2}(1 + \beta) ;$$
$$k_1 = \frac{1}{2}(\alpha_2 - \alpha_1) , \ k_2 = \frac{1}{2}(\alpha_1 + \alpha_2) . \tag{15}$$

Substitution of (14) into expressions (10) and (11) gives

$$n_1 = -\frac{1}{280} C_1^2 - \frac{1}{10} C_1 C_3 - \frac{2}{5} k_1 ; \ n_2 = -\frac{1}{120} C_1 C_2 - \frac{1}{6} C_1 C_4 - \frac{2}{3} k_2 ;$$
$$n_3 = \frac{1}{2520} C_1^2 + \frac{1}{120} C_1 C_3 + \frac{1}{30} k_1 ; \ n_4 = \frac{1}{360} C_1 C_2 + \frac{1}{24} C_1 C_4 + \frac{1}{6} k_2 ; \tag{16}$$
$$p_1 = \frac{1}{60} C_1 k_1 + \frac{1}{6} C_2 k_2 ; \ p_2 = \frac{1}{24} C_2 k_1 + \frac{1}{24} C_1 k_2 + \frac{1}{2} C_3 k_2 - \frac{1}{2} C_4 k_1 .$$

2. Now consider the heat transfer process. Assuming that the temperature drop along the radius of the slot is insignificant for the case of rotational symmetry, this equation can be written in the following dimensionless form:

$$w \, \mathrm{Pe} \frac{dT}{dz} = \frac{d^2 T}{dz^2} . \tag{17}$$

In being made dimensionless, the dimensional temperature is referred to the surface area of the upper subliming disk T_s. As the boundary conditions for Eq. (17), use will be made of the condition of constant sublimation temperature $T = 1$ at $z = 1$ and the heat balance equation $\lambda(dT/dz) = -hq/T_s$ at $z = -1$. Then, the solution of this problem can be written in the form

$$T(z) = 1 - m \int_1^z \exp \left(\mathrm{Pe} \int_1^z w_1(z) \, dz \right) dz . \tag{18}$$

Upon integration, we obtain the temperature distribution along the height of the gap between the disks

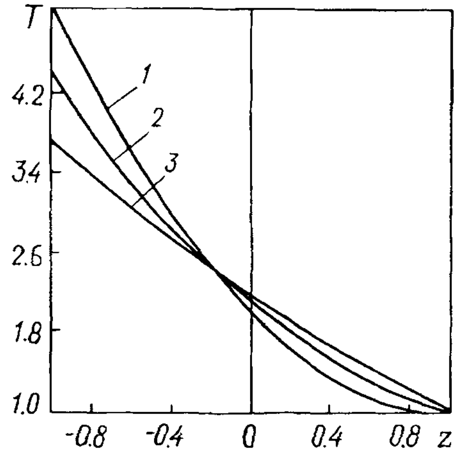

Fig. 2. Temperature distribution in slot between disks: 1, 2, 3) Pe = Re = 0.5, $m = 1$, $\alpha_1 = \alpha_2 = 1$; 1) $\beta = 0$, 2) $\beta = 0.5$, 3) $\beta = 1$.

$$T = 1 - m\left(\exp\left(\mathrm{Pe}\left[-C_1\frac{1}{24} - C_2\frac{1}{6} - C_3\frac{1}{2} - C_4 - \right.\right.\right.$$
$$-\mathrm{Re}\left[C_1^2\frac{1}{40720} + C_1C_2\frac{1}{5040} + C_1C_3\frac{1}{720} + C_1C_4\frac{1}{120} + k_1\frac{1}{180} + \right.$$
$$\left.\left.\left.\left. + k_2\frac{1}{30} + n_1\frac{1}{24} + n_2\frac{1}{6} + n_3\frac{1}{2} + n_4\right]\right]\right)\int_1^z \exp\left(\mathrm{Pe}\left[4C_1\frac{z^4}{24} + \right.\right.$$
$$+ C_2\frac{z^3}{6} + C_3\frac{z^2}{2} + C_4z + \mathrm{Re}\left[C_1^2\frac{z^3}{40720} + C_1C_2\frac{z^7}{5040} + C_1C_3\frac{z^6}{720} + \right.$$
$$\left.\left.\left.\left. + C_1C_4\frac{z^5}{120} + k_1\frac{z^6}{180} + k_2\frac{z^5}{30} + n_1\frac{z^4}{24} + n_2\frac{z^3}{6} + n_3\frac{z^2}{2} + n_4\right]\right]\right)dz\right). \qquad (19)$$

In Fig. 2 one can see the shapes of the temperature distribution in a narrow slot between the disks that were calculated from formula (19) at different values of the injection (suction) coefficient of subliming vapor β from the channel through the porous disks. It can be seen that as β rises, the temperature of the lower heated disk falls. A similar effect is observed as the rotational velocity of the disks increases. This indicates a positive effect of the suction coefficient β and the effect of rotation of the disks α_1 and α_2 on intensification of sublimation in the channel between the disks.

NOTATION

h, half-width of gas gap; q, intensity of heat flux; ω_j, angular velocity of disks, subscript $j = 1, 2$, refers to upper and lower disks, respectively; w_s, sublimation rate; z, r, cylindrical coordinates; ρ, density; p, pressure in gas gap; T_s, sublimation temperature; Re, Reynolds number; $\alpha_j = \omega_j h/w_s$, dimensionless rotation velocity of disk; $\beta = w_1/w_s$, dimensionless injection (suction) coefficient; w_j, injection (suction) rate through porous disk; $\mathrm{Pe} = hw_s/a$, Peclet number; $a = \lambda/(\rho C_p)$, thermal diffusivity; λ, thermal conductivity; C_p, isobaric heat capacity; $m = \mathrm{Pe}r_s/(T_sC_p)$, dimensionless complex; $r_s = q/(\rho w_s)$, sublimation heat.

REFERENCES

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